UNCERTAINTY OF NUMERICAL MODELS FOR PUNCHING RESISTANCE OF RC SLABS

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Abstract

Punching resistance of RC slabs in design practice is based on a model of inclined crack observed in experiments. This simplified engineering model has typical features of empirical models: It is very robust for average case but need not to capture the whole range of structural solutions likely to occur in practice. In this model the effects of transverse reinforcement, or size may not be well represented. Extension to the unsymmetrical stress distribution in the slab-column system due to the moment action is also problematic. Therefore, alternative resistance models based on generally valid principles of continuum mechanics and the state-of-the-art constitutive models are sought. These models are utilizing numerical methods of structural mechanics and are often used for simulation of structural resistance. The numerical simulation became recently an alternative method of design verification. Due to the global nature of nonlinear numerical analysis the design condition is based on a global safety format. However, due to an increased complexity, such models may exhibit a large model uncertainty. The safety formats in the design practice are based on reliability methods, where the safety of design, expressed by a failure probability (or reliability index) is compared with the requirements prescribed by codes or owners. One of the key stones of the safety check is the assessment of the design resistance uncertainty. A probabilistic description of this uncertainty is widely accepted method of reliability assessment. Two sources of uncertainties involved in resistance models are recognized, namely, the aleatory uncertainty due to random properties of input parameters (material parameters, etc.) and the epistemic uncertainty due to a lack of knowledge of models. The presented research deals with the description of uncertainties of numerical models for resistance of punching capacity of slabs. The model uncertainties were identified and subsequently recommendation of a global factor for model uncertainty is proposed.

Keywords: Model uncertainty, Punching, Reinforced concrete slabs, Nonlinear analysis, Numerical simulation, Finite element method, Probabilistic analysis.

1 Introduction

Numerical simulation of structural resistance can be applied to design of reinforced concrete structures using the global safety format, which was recently introduced in the fib Model Code 2010. This topic is in detail described by authors in paper by Cervenka (2013). The design condition is formulated as

$$F_d < R_d \,, \;\; R_d = R_m / \gamma_R \;, \; \gamma_R = \gamma_m \, \gamma_{Rd} \tag{1}$$

In which the F_d , R_d are, respectively, the design values of actions and resistance, and γ_R is the global safety factor. The global safety factor can be further resolved in a product of the partial safety factors due to material γ_m and due to resistance model γ_{Rd} .

For a safe design it is inevitable to introduce a justified concept of model uncertainty. The model uncertainty is defined as the ratio

$$\theta = R_{\exp} / R_{sim} \qquad (2)$$

(resistance found by testing R_{exp} and resistance obtained by a computational model R_{sim} in our case by a numerical simulation) and can be considered as a random variable with a lognormal distribution function described by mean μ_θ and coefficient of variation V_θ. The partial safety factor for model uncertainty γ_{Rd} can be derived for these statistical parameters.

Note that the experimental resistance is considered as a reference (hence a true value). However, the experimental results inevitably include also random effects from other sources, such as materials, testing, etc. Therefore, in order to identify pure model uncertainties, it is essential to reduce such other effects (aleatory uncertainties) to a minimum.

In Euro Code and Model Code the partial factor for model uncertainty in case of nonlinear analysis is recommended by relatively low values. (Eurocode $\gamma_{Rd} = 1.06$ and Model Code $\gamma_{Rd} = 1.0 \div 1.1$) Comprehensive works dealing with model uncertainties (Schlune 2011, JCSS 2001) provide a critical insight into this subject and report that the model uncertainty is much larger. Furthermore it, is also argued that the model uncertainties should be related to failure mode. According to JCSS the model uncertainty related to shear failure is represented by statistical parameters of mean $\mu_\theta = 1.4$ and coefficient of variation $V_\theta = 0.25$. Schlune (2011) found for shear failure of concrete: $\mu_\theta = 0.7 \div 1.0$, $V_\theta = 0.20 \div 0.40$. Such significant differences require a further discussion.

It should be noted that a mean model uncertainty significantly different from unity represents a systematic error of the model. This indicates that such models are not validated and should be either avoided or applied with caution. This is a typical outcome of international contests and bench marks, in which the models cannot be calibrated because they provide blind predictions (For example Collins et al. 1982, Marti 2013). If the poorly validated models are included in the evaluation of model uncertainty they inevitably produce high safety factor. Obviously it is not rational to apply such safety factor to well validated models and the practical value of such approach is questionable.

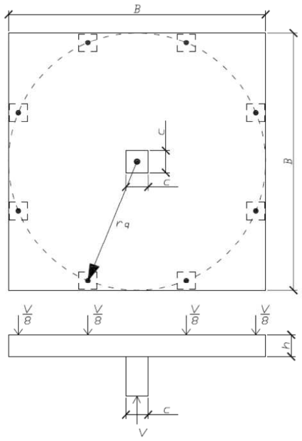
Therefore, the models based on different formulations should be considered separately. The model uncertainty should be related to a certain model (or a constitutive law, a software).

In the view of the above discussion, the present study is related and limited to the specific resistance model based on nonlinear FEM and constitutive model of Atena software.

2 Experiments

Recently a series of tests was performed on slabs in Lausanne under the guidance of Muttoni (Guandilini & Muttoni 2004). This experimental program served also to formulate the punching resistance models recommended in the new *fib* MC2010. The aim of the tests was to investigate the behaviour of slabs failing in punching shear with low longitudinal reinforcement ratios. No transverse reinforcement was applied. The size of the specimens and of the aggregate was also varied to investigate its effect on punching shear. The test series consisted of 11 square slabs with dimensions shown in Fig.1. The dimensions of the tested specimens were of three types: full-size specimens (PG1, PG2b, PG4, PG5, PG10, PG11), double-size specimen (PG3) and half-size specimens (PG6, PG7, PG8, PG9).

During the punching test, the load was increased at a constant speed up to failure. It took approximately 1 hour from the beginning of the loading to failure in punching of specimen. The summary of slab parameters is given in Table 1.



Specimen	*B*	r_q	*h*	*c*
half size	1.5 m	0.752 m	0.125 m	0.13 m
full size	3 m	1.5 m	0.25 m	0.26 m
double size	6 m	2.85 m	0.5 m	0.52 m

Fig. 1 Geometry of tested slabs

Concrete compressive cylinder tests were carried out at the age of slab testing. Compressive strength was the only confirmed concrete parameter. In all specimens except one, hot-rolled steel bars were used. To the contrary- for Specimen PG-5 cold-worked bars were used.

Table 1

Input parameters

	fc **(MPa)**	***fy*** **(MPa)**	***fu*** **(MPa)**	***h*** **(m)**	***d*** **(m)**	***dg*** **(mm)**	**reinforcement**	***ρ*** **(%)**
PG1	27.6	573	656	0.25	0.21	16	Ø20 @100	1.5
PG2b	40.5	552	612	0.25	0.21	16	Ø 10 @150	0.25
PG3	32.4	520	607	0.5	0.456	16	Ø 16 @135	0.33
PG4	32.2	541	603	0.25	0.21	4	Ø 10 @150	0.25
PG5	29.3	555	659	0.25	0.21	4	Ø 10 @115	0.33
PG6	34.7	526	607	0.125	0.096	16	Ø 14 @110	1.5
PG7	34.7	550	623	0.125	0.1	16	Ø 10 @105	0.75
PG8	34.7	525	586	0.13	0.117	16	Ø 8 @155	0.28
PG9	34.7	525	586	0.13	0.117	16	Ø 8 @196	0.22
PG10	28.5	577	648	0.25	0.21	16	Ø 10 @115	0.33
PG11	31.5	570	684	0.25	0.21	16	Ø 16/18 @145	0.75

Legend:

d– distance from compression fibre to centroid of longitudinal tensile reinforcement.

dg – maximum diameter of aggregate.

3 Numerical simulations

3.1 Constitutive models

The punching resistance in this study is modelled by a non-linear analysis based on the finite element method offered by the software Atena, Cervenka et al. (2013). The concrete is described by a fracture-plastic constitutive model (denoted as C3DNonLinCementitious). In this model the tensile behaviour is treated by fracture mechanics and compressive behaviour by plasticity. The crack band approach is used for the strain localization in both, tension and compression. The Rankine criterion with exponential softening is used for cracking. The confinement effect on concrete compressive strength is treated by Menétrey-Willam failure surface. The algorithm can handle the cases when failure surfaces of both models are active, but also when physical changes such as crack closure occur. The model can be used to simulate concrete cracking, crushing under high confinement, and crack closure due to crushing in other material directions. For details see Cervenka and Papanikolau (2008).

The reinforcing bars are modelled by embedded truss elements with a multi-linear stress-strain law. The bond model is based on a bond-slip relationship, in which the model according to the CEB-FIB ModelCode 1990 was used.

The concrete properties are described by the nominal strength of concrete obtained from cylinder compressive tests. All parameters of constitutive law are then obtained using a set of relations (suggested by codes) in order to provide required input data. Such relations form a part of the resistance model and consequently are potential source of model uncertainty. The relations and values used in this study are listed in Table 2, 3 and 4.

Table 2

Relations for additional properties of concrete

Parameter	**Formula**
Cylinder compressive strength	$f_c = -0.85 f_{cu}$
Tensile strength	$f_t = -0.24 f_{cu}^{\frac{2}{3}}$
Initial elastic modulus	$E_c = (6000 - 15.5 f_{cu})\sqrt{f_{cu}}$
Fracture energy	$G_f = G_{f0}\left(\frac{f_c}{10}\right)^{0,7}$ $G_{f0} = 25 N/m, (d_{\max} = 8mm)$ $G_{f0} = 30 N/m, (d_{\max} = 16mm)$ $G_{f0} = 58 N/m, (d_{\max} = 32mm)$

Table 3

Additional concrete parameters derived from concrete compressive strength

	E **(Gpa)**	***f$_t$*** **(MPa)**	***G$_f$*** **(N/m)**
PG1	31.14	2.443	61.06
PG2b	36.32	3.154	79.86
PG3	33.4	2.718	68.31
PG4	33.31	2.707	56.68
PG5	32.09	2.542	53.06
PG6	34.29	2.845	71.67
PG7	34.29	2.845	71.67
PG8	34.29	2.845	71.67
PG9	34.29	2.845	71.67
PG10	31.73	2.496	62.44
PG11	33.03	2.668	66.98

While parameters listed in Table 2 and their values from Table 3 are typical for concrete and are used in many other models, the parameters in Table 4 are specifically related to the Fracture-plastic model in Atena. They represent following features: Volume plastic factor β describes a volume dilation during non-associated plastic deformations within the plastic range; Shear factor relates the fracture energy in mode two (crack sliding) to mode one (crack opening); Compression softening parameter w_d represents a deformation of compression zone after the complete stress fading.

Table 4

Fracture-plastic model parameters

Parameter	Value
Volume dilatation plastic factor [-]	$\beta = 0.5$
Compressive strength in cracked concrete [-]	$r_{c,\lim} = 0.5$
Shear factor [-]	20
Compression softening [m]	$w_d = -0.002$

3.2 Mesh size effect

The size of finite element, i.e., mesh density, has effect on the results of analysis. In case of the stiffness formulation used in the present study this is due to the approximations applied to displacement fields. Therefore the mesh size effect was investigated first. Hexahedral (brick) elements with 8 nodes are used for concrete. A mesh size study was performed for specimen PG2b for three different mesh sizes with 3,5 and 7 cube bricks through the slab thickness. The mesh size was the same in the whole slab. See example of mesh with 5 elements in Fig.3. The study confirmed that mesh with 5 elements was fine enough and a further refinement did not bring a significant improvement. Therefore, the model with 5 element through the thickness was used in the following model uncertainty study.

The symmetry was utilized in order to reduce the model size to one quarter of the geometry. The symmetry assumption reduction can also have an influence on resistance in case of a brittle failure mode. If we expect a chain-type of failure only one weak spot occurs in a full model, while in our symmetrical model at least four weak spots are included. This question was investigated by comparing a model reduced to one quarter with a full model in which a minor difference was observed and consequently the quarter model was used.

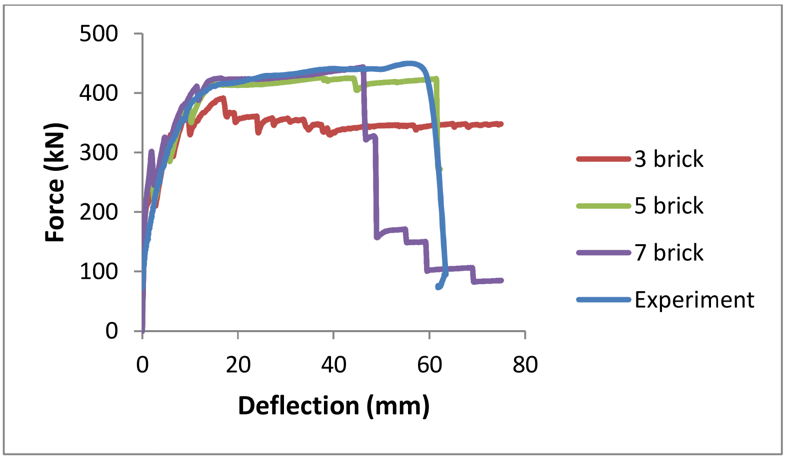


Fig. 2 Slab PG2b - Mesh sensitivity test (slab thickness with 3, 5 and 7 elements)

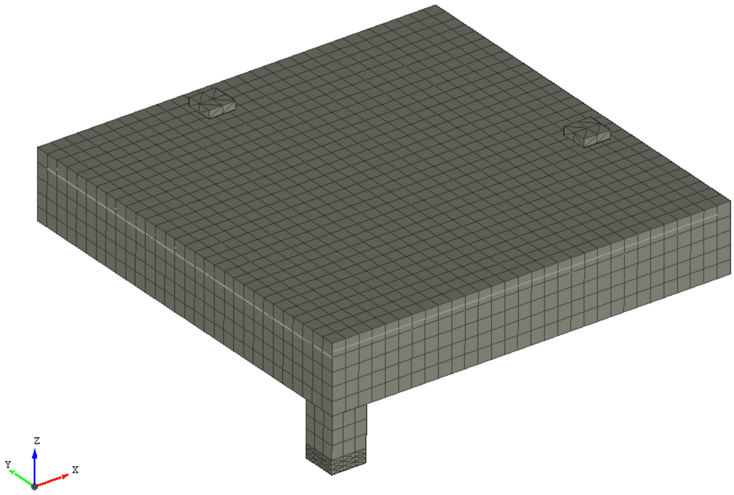


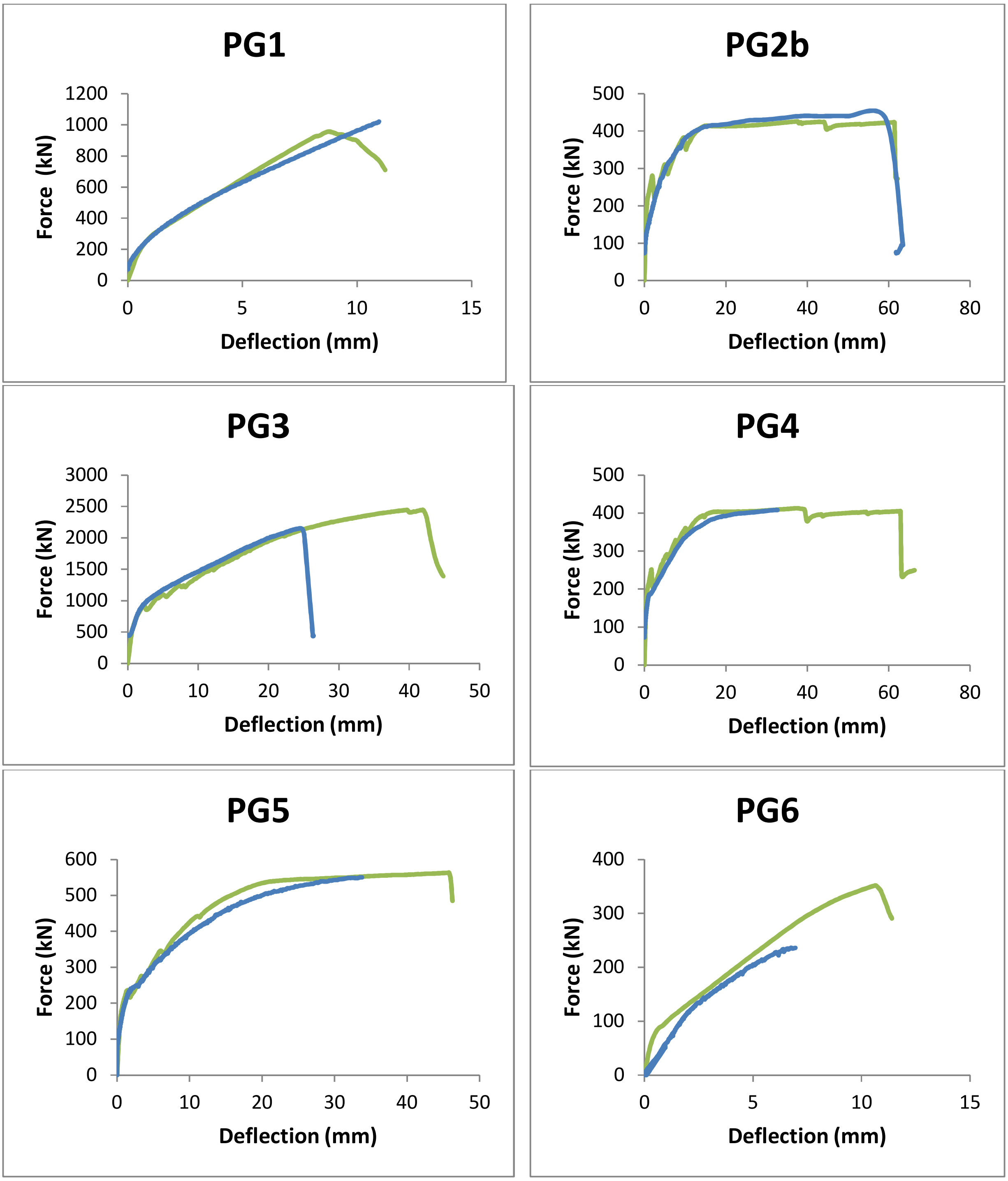
Fig. 3 Typical mesh (slab thickness divided into 5 brick elements)

3.3 Solution method

The load was applied by imposing a prescribed displacement at the column head and the resistance was found as a reaction at the loading point. The sliding vertical supports were applied at support plates. Newton-Raphson iterative solution was used with a tangent stiffness and admissible error of residual forces at 0.01 (1%). Before reaching the ultimate strength the prescribed error tolerance was satisfied in majority of steps. After the peak the error exceeded the limit.

3.4 Results of simulation

Load-displacement diagrams of slab punching test simulations compared with experiments are shown in Fig.4.


PG1
Force (kN)
Deflection (mm)
PG2b
Force (kN)
Deflection (mm)
PG3
Force (kN)
Deflection (mm)
PG4
Force (kN)
Deflection (mm)
PG5
Force (kN)
Deflection (mm)
PG6
Force (kN)
Deflection (mm)

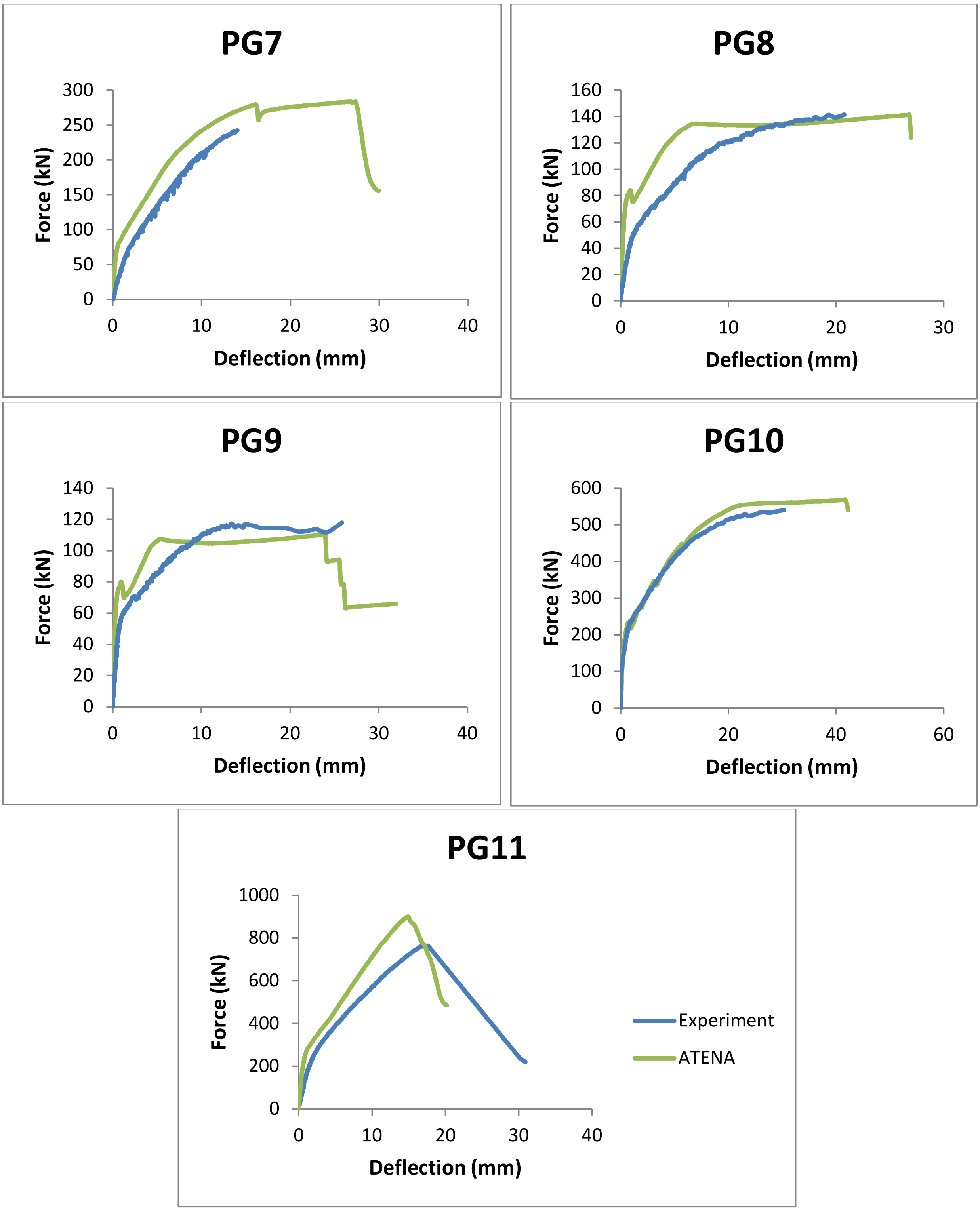


Fig. 4 ATENA numerical simulations compared with experimental results

The specimens covered a wide range of punching behaviour (see Fig.1 and Table 1). Reinforcing ratio varied from 0.25 to 1.5 % and the thickness from 0.125 to 0.5 m. In all specimens the failure mode had a quality of punching failure with formation of a cone-like surface. In two tests with low reinforcement ratio, PG2b and PG4 the reinforcement failed in tension. In other cases reinforcement yielded during the punching process. In highly reinforced slabs the reinforcement did not yielded and the maximum resistance was marked by a concrete failure. In all cases (except the excluded slab PG6) the analysis provided a good simulation of experimental behaviour.

The size effect could be directly observed by comparing specimens PG5 and PG3, with identical reinforcing ratio but size scale factor 2, where the large slab has shown more brittle response.

Interesting results are shown in tests PG2B and PG4. Both specimens are identical except of the aggregate size and slight difference in concrete strength. These properties, including the aggregate size, were duly considered in the input data. The analytical curves for both simulations are almost identical. However, the experimental curve of PG4 is terminated earlier then in PG2b. This difference is not discussed in the test report, but it can be speculated, that the small aggregate concrete had more brittle response then in the analysis. In any case a simulation of two almost identical cases provide a valuable contribution for a the assessment of model uncertainty.

3.5 Exclusion of outliers

In case of test PG6 strong discrepancy between analytical and measured data were observed in the early loading stage. This could not be rationally explained and could indicate some unknown difference between the model and the test. It is known from other validations that simulation should well capture the initial elastic response. This can be observed in other tests in this series. From this reason the test of specimen PG6 it was excluded from the statistical set.

4 Model uncertainties

4.1 Probabilistic analysis

Model uncertainty θ according to Eq.(2) was calculated for each test as listed in Table 4 and allowed to determine the statistical parameters of model uncertainty, namely mean μ_θ and coefficient of variation V_θ. The mean value close to 1 indicates a good model quality. These data can serve for an assessment of partial safety factor of model uncertainty as will be shown later.

4.2 Statistical sensitivity to input parameters

The assumption of a random nature of the model uncertainty can be investigated by correlation between the model uncertainty θ and model parameters *X*. The correlation coefficients for relevant parameters are listed in Table 6.

Table 5

Model uncertainties

	Ultimate strength (kN)		**θ model uncertainty**
	ATENA	**Experiment**	
PG1	957.5	1020.9	1.066
PG2b	425.6	440.7	1.035
PG3	2442	2153	0.882
PG4	412.6	407.9	0.989
PG5	563.1	550	0.977
PG7	283.9	240.9	0.849
PG8	141.3	141.4	1.001
PG9	110.5	118	1.068
PG10	568.3	540.4	0.951
PG11	898.8	763.3	0.849
mean μ_θ			**0.966**
coefficient of variation V_θ			**0.086**

Table 6

Correlation coefficients between input parameters *X* and model uncertainty θ

X	**Correlation coefficient θ(*X*)**	***X***	**Correlation coefficient θ(*X*)**
f_c	0.088	ρ	-0.009
f_y	-0.087	E	0.040
h	-0.282	f_t	0.077
d	-0.276	G_f	0.077

It may be concluded that almost none dependency was observed for the material parameters. This confirmed the robustness of the model under consideration. Nevertheless it should be noted that the range of input parameters was relatively small. Certain small dependency was found for dimension parameters *h* and *d*. However this might be due to the fact that only one test for large size was investigated.

Furthermore, observing that the failure mode (due to concrete, or due to reinforcement) is strongly dependent on the reinforcing ratio we can extend the low sensitivity also to failure mode. This confirms the approach adopted in this study, in which the model uncertainty is investigated for all failure modes in one set.

4.3 Safety factor of model uncertainty based on log-normal distribution

Assuming a lognormal distribution the safety factor due to model uncertainty can be obtained for a chosen reliability index and known statistical parameters:

$$\gamma_{Rd} = \frac{\exp(\alpha_R \beta V_\theta)}{\mu_\theta} \tag{3}$$

The weight factor for resistance $\alpha_R = 0.4 \times 0.8 = 0.32$, where 0.4 is a factor for non-dominant variable and 0.8 is a weight factor for resistance (according to JCSS 2001). The reliability index can be typically chosen as $\beta = 3.8$. Entering these factors and statistical parameters derived in Table 5 into Eq.(3) the value of the model safety factor is obtained as $\gamma_{Rd} = 1.149$ for the investigated slab punching set.

It is interesting to compare the probabilities of admissible failures in the above probabilistic concept. For chosen reliability index combined with a non-dominant weight factor the probability is $P_f = 0.112$. This is the probability of a faulty resistance model described by model uncertainty factor γ_{Rd}, Eq.(3).

For the same reliability index the safety factor due to material uncertainty safety factor γ_m in Eq.(1) corresponds to the failure probability $P_f = 0.001$ as shown in MC2010. (More information about safety formats and determination of design resistance for material uncertainty see paper by Cervenka 2013.) This means that we admit a lower safety margin for model uncertainty (by factor 100) then for the material uncertainty. Such different probabilities are due to the introduction of non-dominant description of model uncertainty. In the view of the above it is recommended to apply the non-dominant factor with caution.

4.4 Safety factor of model uncertainty based on Student distribution

The above probabilistic analysis is based on the assumption of a sufficiently extensive set of statistical data. However, in practical cases the model is often validated on a limited number of tests. In the present study we consider only 10 experiments. For limited set of data it may be more justified to use a Student distribution which respects this fact. Student distribution depends on number of samples *n*. For large number of samples the Student distribution is approaching to a originally assumed log-distribution.

$$\gamma_{Rd} = \frac{1}{\mu_\theta \exp(t_{p=0.112}(n-1) \times V_\theta)} \tag{4}$$

where $t_{p=0,112}(n-1)$ is the quantile of Student distribution for probability *p*=0.112 with *n*-1 samples. Using the statistical parameters found in Table 4 and number of samples *n*=10 the safety factor according to Eq.(4) is $\gamma_{Rd} = 1.158$. It may be concluded, that the value is very close to the one based on the log-normal distribution and thus this method does not change the resulting safety factor and the number of samples is apparently sufficient.

5 Conclusions

Model uncertainty should be reflected in a safety assessment of design resistance. When using numerical simulations based on nonlinear finite element analysis it is suggested to use a validation by experiments for determination model uncertainty. The safety factor of model uncertainty should be limited to a specific numerical models or software.

The authors performed a probabilistic analysis of model uncertainty for punching failure of reinforced concrete slabs based on resistance model in software ATENA and validation by experimental data of Muttoni et al. They found the global safety factor for model uncertainty $\gamma_{Rd} = 1.15$.

If only a low number of experiments is available for validation, let say less than 10, the Student distribution should be used for the determination of safety factor of model uncertainty.

Extension to other resistance models based on another software with different underlying constitutive models would require additional adequate validation. Further research is needed for assessment of model uncertainty for other types of failure.

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